

# Width difference of the $B_s$ mesons from lattice QCD

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We extend our previous studies to calculate the  $B$  meson  $B$  parameters  $B_B$  and  $B_S$  on the lattice, and present an update of the results for the  $B_s$  meson width difference. We perform an extensive study of systematic errors in the quenched calculation of the  $B$ -parameters, and find that the systematic errors are in good control using the NRQCD action for heavy quark. We also report our preliminary results from unquenched simulations.

## 1 Width difference $\Delta\Gamma_{B_s}$

The width difference  $\Delta\Gamma_{B_s}$  in the  $B_s - \bar{B}_s$  system opens a new possibility to measure the CKM angles and to search for new physics, once it is measured and found to be sizable<sup>1</sup>. Theoretical prediction of  $\Delta\Gamma_s$  is, therefore, desirable, and the most reliable calculation has been obtained using the Heavy Quark Expansion under an assumption of the quark-hadron duality<sup>2,3</sup>, and a recent summary is given in<sup>4</sup>. In this calculation, nonperturbative inputs are necessary for the  $B$  meson  $B$  parameters  $B_B$  and  $B_S$ , for which the lattice QCD may provide first-principles calculation starting from the QCD lagrangian. In this report, we present an update of the lattice calculation of  $B_B$  and  $B_S$  by the JLQCD collaboration. Our previous works are published in<sup>5,6,7</sup>.

The main contribution to  $\Delta\Gamma_{B_s}$  comes from the  $c\bar{c}$  final state, to which both  $B_s$  and  $\bar{B}_s$  can decay, and the theoretical expression obtained using the Heavy Quark Expansion<sup>a</sup> was given by Beneke *et al.* as<sup>2,3</sup>

$$\Delta\Gamma_{B_s} = \frac{G_F^2 m_b^2}{12\pi M_{B_s}} |V_{cb}^* V_{cs}|^2 \times [c_L(z)\langle\mathcal{O}_L\rangle + c_S(z)\langle\mathcal{O}_S\rangle + c_{1/m}(z)\delta_{1/m}], \quad (1)$$

where  $c_L(z)$ ,  $c_S(z)$  and  $c_{1/m}(z)$  are known functions of  $z = m_c^2/m_b^2$ . The four-quark operators  $\mathcal{O}_L = \bar{b}(1 - \gamma_5)\gamma_\mu s \bar{b}(1 - \gamma_5)\gamma_\mu s$  and  $\mathcal{O}_S = \bar{b}(1 - \gamma_5)s \bar{b}(1 - \gamma_5)s$  are evaluated for  $B_s$  and  $\bar{B}_s$  mesons as initial and final states respectively. Normalizing with the total width  $\Gamma_{B_s}$ , one arrives at

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \frac{16\pi^2 B(B_s \rightarrow X e \nu)}{g(z)\tilde{\eta}_{QCD}} \frac{f_{B_s}^2 M_{B_s}}{m_b^3} |V_{cs}|^2 \times \left[ G(z) \frac{8}{3} B_B(m_b) + G_S(z) \frac{5}{3} \frac{B_S(m_b)}{\mathcal{R}(m_b)^2} \right]$$

<sup>a</sup> As pointed out by Falk at the conference, the operator product expansion could be dangerous for this quantity, because the energy release is not large enough for the  $c\bar{c}$  final states. The quark-hadron duality could also be questionable, as the final states are saturated by a limited number of exclusive modes. Unfortunately, no rigorous method is known to estimate the violation of these assumptions.

$$\left. + \sqrt{1 - 4z} \delta_{1/m} \right] \quad (2)$$

$$= \left( \frac{f_{B_s}}{230 \text{ MeV}} \right)^2 \times \left[ 0.007 B_B(m_b) + 0.132 \frac{B_S(m_b)}{\mathcal{R}(m_b)^2} - 0.078 \right], \quad (3)$$

where  $B_B(m_b)$  and  $B_S(m_b)$  are  $B$ -parameters defined as

$$B_B(\mu) = \frac{\langle \bar{B}_s | \mathcal{O}_L(\mu) | B_s \rangle}{\frac{8}{3} f_{B_s}^2 M_{B_s}^2}, \quad (4)$$

$$B_S(\mu) = \frac{\langle \bar{B}_s | \mathcal{O}_S(\mu) | B_s \rangle}{-\frac{5}{3} f_{B_s}^2 M_{B_s}^2} \times \mathcal{R}(\mu)^2, \quad (5)$$

and

$$\mathcal{R}(m_b) \equiv \frac{\bar{m}_b(m_b) + \bar{m}_s(m_b)}{M_{B_s}} = 0.81(3). \quad (6)$$

is a ratio of matrix elements of heavy-light axial current and pseudo-scalar density<sup>5</sup>.<sup>b</sup> Other notations may be found in<sup>3</sup>. We note that for  $(\Delta\Gamma/\Gamma)_{B_s}$  the matrix element of  $\mathcal{O}_S$  gives dominant contribution while the effect of  $\mathcal{O}_L$  is negligible. The  $1/m$  correction  $\delta_{1/m}$  is expressed in terms of matrix elements of higher dimensional  $\Delta B=2$  operators, and is found to be sizable (and destructive) by an evaluation using the vacuum saturation approximation<sup>2</sup>.

To obtain the prediction for  $(\Delta\Gamma/\Gamma)_{B_s}$ , nonperturbative inputs are necessary for the decay constant  $f_{B_s}$  and the  $B$ -parameters  $B_B$  and  $B_S$ . The quenched lattice calculation of the  $B$  meson decay constant is quite stable over years in the sense that results from many groups using different lattice actions agree, and a recent summary is  $f_{B_s} = 195(20) \text{ MeV}$ <sup>9</sup>. Several groups are recently performing unquenched calculations that include the effect of light quark loops. Although the calculation is computationally much demanding, the unquenched QCD simulation is being realistic on recent dedicated machines or on commercial supercomputers. Recent results suggest that the decay constants become higher with the

<sup>b</sup> The same notation  $\mathcal{R}(m_b)$  is used for a different quantity in<sup>8</sup>.

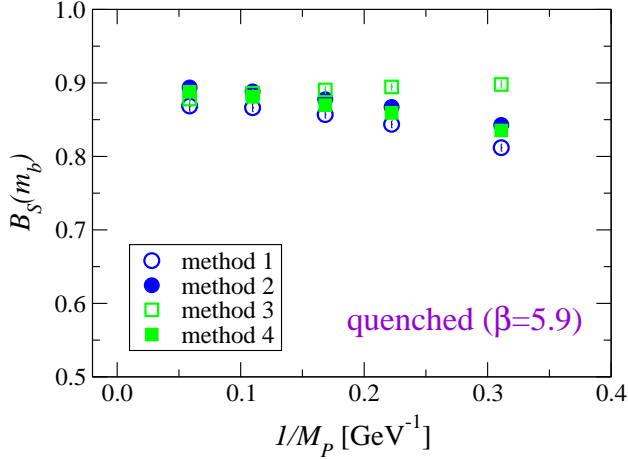


Figure 1: Quenched results for  $B_S(m_b)$  as a function of  $1/M_P$  at  $\beta=5.9$ . Results with four different methods are plotted.

dynamical quark effect, and the most recent summary by Bernard at Lattice 2000 is  $f_{B_s} = 230(30)$  MeV<sup>10</sup>, which we use in this work.

## 2 Lattice calculation of $B$ parameters

Present work is an extension of previous calculations by Hiroshima group<sup>5,6</sup>, and preliminary results are already reported in<sup>7,11</sup>.

We use the lattice NRQCD action for heavy quark including all  $1/M$  corrections consistently. Therefore, no extrapolation in the heavy quark mass is necessary in contrast to the work by APE collaboration<sup>8</sup>, in which they employ the relativistic action for relatively light heavy quark and extrapolate the results to  $m_b$ .

We perform a study of systematic errors on quenched ( $N_f=0$ ) lattices. We calculate the  $B$ -parameters in four different methods that have different systematic errors coming from higher orders in the  $1/M$  and  $\alpha_s$  expansions, in order to see the associated systematic uncertainties.<sup>c</sup> In addition, the simulations are repeated at three lattice spacings to see the systematic error depending on lattice spacing  $a$ .

Results for  $B_S(m_b)$  are plotted in Figure 1, from which we can observe that the disagreement among the different methods becomes larger for larger  $1/M$ . That behavior is expected, because the  $1/M$  expansion is truncated at first order in our calculation. Nevertheless the result at the  $B_s$  meson mass ( $1/M_P \simeq 0.19$  GeV<sup>-1</sup>) is under reasonable control. A similar plot is shown in Figure 2 for three lattice spacings in the quenched approximation. We find a good agreement among the results from different lattices, which indicates that the system-

<sup>c</sup> For a detailed description of the four methods, see<sup>7</sup>.

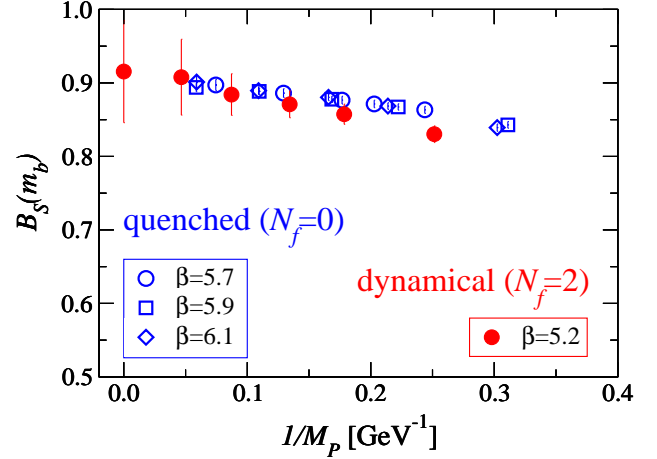


Figure 2:  $B_S(m_b)$  as a function of  $1/M_P$  at three lattice spacings. The method 2 is used. Unquenched result is also plotted (filled circles).

atic error associated with the lattice discretization is well controlled.

At this conference, we presented a new result from unquenched simulations. As one of the major projects of the JLQCD collaboration, we are performing a two-flavor (dynamical  $u$  and  $d$  quarks) QCD simulation with a non-perturbatively  $O(a)$ -improved quark at  $\beta=5.2$ ,  $c_{sw}=2.02$  on a  $20^3 \times 48$  lattice, which corresponds to the lattice spacing  $a \simeq 0.1$  fm. A preliminary report of this simulation may be found in<sup>12</sup>. On this lattice, we are calculating the  $B$ -parameters and a very preliminary result is plotted in Figure 2. Although the result is slightly lower than the quenched data, the difference is statistically not significant, and our conclusion at this stage is that the effect of dynamical quarks is not sizable for the  $B$ -parameters.

Our (preliminary) results for  $B$ -parameters in the quenched ( $N_f=0$ ) and unquenched ( $N_f=2$ ) calculations are

$$B_B(m_b) = \begin{cases} 0.85(2)(8) & (N_f = 0) \\ 0.83(3)(8) & (N_f = 2) \end{cases}, \quad (7)$$

$$B_S(m_b) = \begin{cases} 0.87(1)(9) & (N_f = 0) \\ 0.84(6)(8) & (N_f = 2) \end{cases}. \quad (8)$$

The systematic errors are evaluated using the variation among four different methods as a guide. Using (3) the result for the width difference is obtained as

$$\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = 0.097^{+0.014}_{-0.035} \pm 0.025 \pm 0.020 \pm 0.016, \quad (9)$$

where errors are from residual scale  $\mu$  dependence remaining in the calculation of  $G(z)$  and  $G_S(z)$  in (3),  $f_{B_s}$ ,  $B_S$ , and the uncertainty in the  $1/m$  corrections, respectively. The central value 0.097 is somewhat smaller than

our previous estimate  $0.107(26)(14)(17)^6$  due to a smaller input for  $f_{B_s}$ , which was previously  $245(30)$  MeV. The effect of unquenching is less significant ( $-6\%$ ). The experimental results from LEP and SLD were summarized by Boix at this conference<sup>13</sup> as  $(\Delta\Gamma/\Gamma)_{B_s} = 0.16^{+0.08}_{-0.09}$ .

One of the major systematic errors in (9) comes from the uncertainty in  $f_{B_s}$ , which may be avoided by considering a ratio  $(\Delta\Gamma/\Delta M)_{B_s}$ , once  $\Delta M_s$  is measured<sup>3</sup>. We obtain

$$\begin{aligned} \left(\frac{\Delta\Gamma}{\Delta M}\right)_{B_s} &= \frac{\pi}{2} \frac{m_b^2}{M_W^2} \left| \frac{V_{cb}^* V_{cs}}{V_{tb}^* V_{ts}} \right|^2 \frac{1}{\eta_B(m_b) S_0(x_t)} \\ &\times \left[ \frac{8}{3} G(z) + \frac{5}{3} G_S(z) \frac{B_S(m_b)}{B_B(m_b)} \frac{1}{\mathcal{R}(m_b)^2} \right. \\ &\quad \left. + \frac{\sqrt{1-4z} \delta_{1/m}}{B_B(m_b)} \right], \\ &= \left( 0.20 + 6.00 \frac{B_S(m_b)}{B_B(m_b)} - 2.85 \right) \times 10^{-3} \\ &= (3.5^{+0.4}_{-1.3} \pm 0.6 \pm 0.6) \times 10^{-3}, \end{aligned} \quad (10)$$

where errors come from  $\mu$ ,  $B_S/B_B$ , and the  $1/m$  corrections, respectively.

### 3 Comparison with other approaches

There are two other approaches to calculate the  $B$  meson  $B$ -parameters on the lattice.

One is the HQET approach, in which the  $b$  quark is treated as a static color source, and it is a naive limit  $M \rightarrow \infty$  of the NRQCD action. Giménez and Reyes calculated  $B_B$  and  $B_S$  in this method<sup>14,15</sup>, in which an unquenched calculation is also performed (but with an unimproved action).

The other is the relativistic approach, in which the relativistic lattice action is used for heavy quark. In order to avoid large discretization error growing as a power of  $aM$ , one cannot simulate the  $b$  quark and has to treat relatively light heavy quark with mass around 1–2 GeV. An extrapolation in the heavy quark mass is then attempted. The crucial point for this method is to find a heavy quark mass region where  $M$  is small enough so that the discretization error of  $O((aM)^2)$  is under control, and at the same time  $M$  is large enough to justify the use of heavy quark expansion in the extrapolation. In general it may not be easy to identify such a region with confidence, and the applicability of the method should be studied carefully depending on the quantities to calculate. Calculation of the  $B$ -parameters in this method has been presented by APE<sup>8</sup> and UKQCD<sup>16</sup> collaborations.

In Figure 3 we compare our results with other approaches. While our data cover the physical region  $1/M_P$

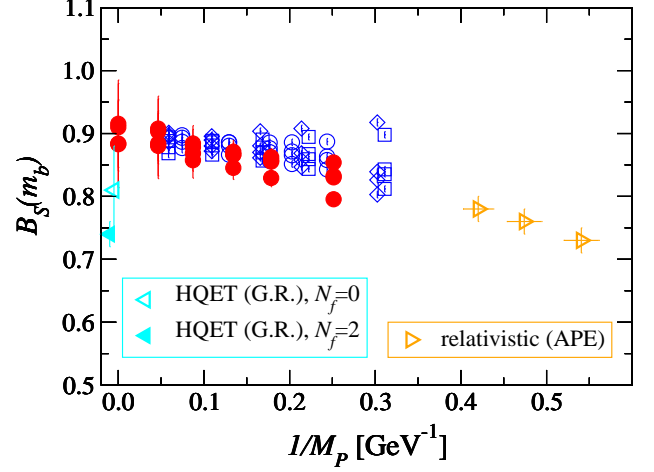


Figure 3:  $B_S(m_b)$  from different approaches. Our results are shown by open symbols ( $N_f=0$ ) and solid circles ( $N_f=2$ ).

$= 0.05$ – $0.3$   $\text{GeV}^{-1}$ , the HQET results are plotted at the static limit  $1/M_P = 0$ , and the relativistic results are shown in the charm quark mass region  $1/M_P = 0.4$ – $0.5$   $\text{GeV}^{-1}$ . Although we did not attempt to fit the whole data with a single curve, it seems that the results of different approaches agree reasonably well. It suggests that the lattice calculation of  $B_S$  is in good shape, even though the individual method might have its own systematic uncertainty.

Besides the calculation of  $B$ -parameters, the APE collaboration proposed to express  $\Delta\Gamma_{B_s}$  using  $\Delta M_d$  as a normalization<sup>8</sup>

$$\begin{aligned} \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} &= K \left( \tau_{B_s} \Delta M_d \frac{M_{B_s}}{M_{B_d}} \right) \left( \frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_d}^2 \hat{B}_{B_d}} \right) \left| \frac{V_{ts}}{V_{td}} \right|^2 \\ &\times \left[ G(z) + G_S(z) \frac{5}{8} \frac{B_S(m_b)}{B_B(m_b)} \frac{1}{\mathcal{R}(m_b)^2} \right. \\ &\quad \left. + \frac{3}{8} \frac{\sqrt{1-4z} \delta_{1/m}}{B_B(m_b)} \right], \end{aligned} \quad (11)$$

and quoted a much smaller result  $0.047 \pm 0.015 \pm 0.016$ .  $K$  is a known factor, and an experimental value may be used for  $(\tau_{B_s} \Delta M_d \frac{M_{B_s}}{M_{B_d}})$ . In the ratio  $\frac{f_{B_s}^2 \hat{B}_{B_s}}{f_{B_d}^2 \hat{B}_{B_d}}$  the bulk of systematic error should cancel in the lattice calculation, and the result has much smaller uncertainty. The problem in this normalization is, however, that the large uncertainty enters implicitly through the (ratio of) CKM elements  $|V_{ts}/V_{td}|^2$ , for which they use a value obtained from a global fit of the CKM elements. We do not take this strategy, because the several sources of (theoretical) systematic errors are hidden in the CKM fit, and it makes the estimate of systematic uncertainties less transparent.

## 4 Requirements for further improvement

In order to improve the accuracy of the prediction (9), better determination of  $f_{B_s}$  is important. For the unquenched lattice calculation, more work is necessary to reach the situation in the quenched case, where consistency is checked among several different approaches by many groups. The similar work is necessary for the  $B$ -parameters, for which only a few unquenched calculations are available including ours.

In (3), it is evident that the  $1/m$  correction gives a large negative contribution to the width difference. Therefore, its reliable evaluation is quite important. At present, it is estimated using the vacuum saturation approximation, and thus nonperturbative calculation on the lattice is desirable. Although it is hard to determine the (perturbative) matching coefficients for higher dimensional operators, the lattice calculation should still be useful to have an idea how reliably the  $1/m$  corrections are estimated.

## 5 Conclusions

The JLQCD collaboration started a lattice calculation of  $B_B$  and  $B_S$  including the effect of sea quarks. A preliminary result shows that the quenching effect is not substantial for these quantities.

We show that the calculations with several different methods on three lattice spacings show a reasonable agreement in the quenched approximation. It indicates that, using the lattice NRQCD, the systematic errors are under control.

The results for  $(\Delta\Gamma/\Gamma)_{B_s}$  still have large uncertainty. Better estimation of the nonperturbative inputs as well as the  $1/m$  corrections will be necessary to improve the accuracy.

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